The distribution of flow divider offset, branch angle, and angular asymmetry in a collection of 70 human aortic bifurcations are reported. A computer algorithm was used to determine these parameters objectively. The mean values with standard deviations and ranges are: 1) magnitude of flow divider offset = 1.4 ± 1.4 mm (range = 0.04 to 8.1 mm); 2) branch angle = 35.0° ± 11.1° (range = 10.4° to 61.3°); and 3) angular asymmetry = 15.3° ± 12.9° (range 0.2° to 56.1°). Since these parameters have been shown to have a significant influence on hemodynamic shear, their variability suggests a corresponding variability in the fluid dynamic stresses to which the bifurcations were exposed in life. (Arteriosclerosis 6:109–113, January/February 1986)

In a recent study using human arterial casts, Friedman et al.1 identified four geometric features of aortic bifurcations whose variation from one individual to another resulted in significant variability in the hemodynamic shear stress at particular sites on the vessel wall. The four specific geometric features were:

1. A large angle of one or both daughters with the aortic axis;
2. A flow divider that did not lie on the axis of the terminal aorta, called flow divider offset;
3. An iliac artery that made a large angle with the aortic axis while the other one closely followed that axis;
4. A gentle intrusion of the arterial wall into the flow as the flow divider was approached.

The first three of these characteristics resulted experimentally in regions of inordinately low shear. In a related investigation, we demonstrated a negative correlation between interfacial shear rate and intimal thickness.2 In view of the association between intimal thickening and arterial pathology, this paper focuses on branch angle, angular asymmetry, and flow divider offset.

The possible importance of branch angle has been recognized and discussed in earlier literature. Saltissi et al.3 and Sharp et al.4 related arterial branch angle to the incidence of occlusive disease. Hutchins et al.5 Raso et al.6 and Lee et al.7 have published anatomical studies of arterial features including branch angle. Other researchers have studied the various fluid dynamic effects of branching because of their presumed importance to atherogenesis.8-14 On the other hand, neither flow-divider offset nor angular asymmetry have received similar attention. The purpose of this paper is to call attention to the variability in these other parameters and to present their distributions and those of branch angle among human aortic bifurcations.

We carried out a retrospective study to determine objectively the distributions of angles and offsets within a collection of 70 aortic bifurcations. We created computer routines that calculated these quantities using as input the wall contours obtained from frontal-plane radiographs of these arterial segments.

Methods

The aortic bifurcations were obtained at autopsy. Of the 70 specimens, 42 were from women and 28 were from men. The mean age of the women was 57.3 ± 14.8 years (SD), with a range of 21 to 80 years. The mean age of the men was 57.3 ± 18.1 years, with a range of 16 to 81 years. The arteries were cannulated and perfused with a 4% formaldehyde solution that was maintained under a hydrostatic pressure head equivalent to the blood pressure recorded during life. The pressure was maintained...
for 2 or 3 days by recirculating the formaldehyde with a vacuum pump and one-way valve arrangement that has proved satisfactory for long-term distentions. It is important to fix the tissue under pressure in order to restore the vessel dimensions to in vivo values. In so doing it is likely that other geometric features are also returned to in vivo proportions. While every attempt was made during fixation to restore the vessel to its topography in the body, the loss of support from the underlying axial skeleton and possible loss of adventitial tethering may have introduced slight changes in the anatomy. After completion of fixation, the vessels were injected with a water-soluble radiographic contrast medium, and stereoscopic radiographs were prepared in the sagittal and frontal planes. Only the frontal-plane radiographs were used for this investigation. Although the aortoiliac bifurcation is occasionally nonplanar, its curvature in the sagittal plane is small and is expected to have only a minor effect on the derived distributions.

Computer routines were written to derive systematically flow divider offsets, branch angles, and angular asymmetries. The development of the algorithm for the computation was guided by subjective evaluations of the geometric parameters of several branches. The radiograph was mounted on a digitizing table with the right side at the top (Figure 1). By using the radiograph, the bifurcation was digitized in two dimensions (X and Y) approximately every 5 mm in the longitudinal direction, with a precision of 0.1 mm. The X coordinate was roughly parallel to the parent axis as determined by eye. Next, cubic spline functions were derived for the upper and lower outer walls of the branch, from 3 to 4 cm proximal to the flow-divider tip to 3 to 4 cm distal; spline functions for the inner walls were determined 3 to 4 cm into the branches. By using the spline functions, the routine stepped upstream from the flow divider in 0.1 mm increments until the throat (minimum vessel diameter) was found. In this instance, diameter was defined as the "frontal-plane vessel dimension perpendicular to direction of incrementation" (i.e., the direction of the visual parent axis). After continuing upstream one-quarter of the diameter at the throat, the midpoints of the parent vessel were determined every 0.1 mm. These points were fit by least squares analysis to a straight line, which was taken to be the axis of the terminal aorta. The aortic axis was defined in this way so that it was independent of the geometric parameters that were sought. The daughter axes were determined from similar least-squares fits over a distance from their origin equal to the daughter branch diameter at the flow-divider tip. Diameter used here refers to the difference between the Y values at the "right" and "left" walls of the vessel; the midpoint is the average of these values.

As shown in Figure 1, flow divider offset was defined as the value of the Y intercept of the extrapolated aortic axis. By letting \( \phi_r \) and \( \phi_l \) be the angles between the right and left daughter axes and the aortic axis, respectively, branch angle was defined as \( \phi_r + \phi_l \). Angular asymmetry was the difference between the angles, \( \phi_r - \phi_l \). These parameters were determined for 70 aortic bifurcations of the original 85 aortic bifurcations in the collection. No throat was found for six of these; the throats in eight others were too close to the proximal end of the radiograph to permit the aortic axis to be found; and in one vessel, the iliac segment was too short to allow the iliac axis to be determined. The concept of a throat was very useful, as indicated in the previous paragraph. The fact that in six vessels no throat was found only means that the aortic segment was too short to determine its location, since the diameter was decreasing in all six.

**Results**

On symmetry grounds, we expected both the mean and the most probable offset to be small. This was true for the mean but not for the most probable offset. The mean offset for all vessels was 0.02 ± 0.75 mm (sd). Of the 70 bifurcations, there were 37 bifurcations in which the divider tip was offset to the right with an average value of 1.3 ± 1.3 mm (sd). For the 33 vessels in which the offset was to the left, the mean was 1.4 ± 1.8 mm (sd). The respective distributions of those vessels with offsets to the right and left were not significantly different. The tendency of flow dividers to the offset is illustrated in Figure 2, which presents the distribution of the magnitude of all offsets. Note that the most probable offset was about 0.75 mm. The mean magnitude of the offsets was 1.4 ± 1.4 mm (sd).
The branch angle distribution is shown in Figure 3. The mean branch angle was 35.0°, with a standard deviation of 11.1°. The average angle between the axes of the terminal aorta and the right iliac artery was 18.6° ± 11.0° (SD); the mean angle between the left iliac artery and the aorta was 16.4° ± 11.7° (SD). There was only a slight skewness toward larger angles, as can be seen from the histogram. The range of the branch angles was 10.4° to 61.3°. As discussed below, the broad range of branch angles can result in major variations in the hemodynamic stresses on the arterial walls. Certainly, there will be a large difference between the mural stresses in vessels whose branch angles are at opposite ends of the distribution.

The angular asymmetry distribution is given in Figure 4. The mean value was 2.2° with a standard deviation of 19.8°. The maximum magnitude was 56.1°. As might be surmised, angular asymmetry is correlated with flow divider offset (correlation coefficient = 0.69, p < 0.0001). For example, the vessel with the largest angular asymmetry (−56.1°) had the largest offset (−8.1 mm), and the case with the second largest value (52.1°) had the fourth largest offset (4.4 mm). Usually, the flow divider was offset toward the daughter branch that makes the larger angle with the aortic axis. This was the case for the branch whose tracing was used to construct Figure 1.

For each of the geometric parameters, a correlation analysis was made with age and with one another. There was no significant correlation between the geometric values or their magnitudes with age. Similarly, analysis of the male and female subgroups yielded no significant correlation with age. The only
correlation of significance was the one already cited between offset and angular asymmetry. Analysis of the statistics on the male–female subgroups (Table 1) shows that the magnitudes of offset and angular asymmetry were significantly different for men and women.

### Discussion

Our results for branch angles are similar to those reported in the literature. From 184 normal adult abdominal aortograms, Lee et al.⁷ obtained a mean branch angle of 40.⁸ with a standard deviation of 11.⁷. Even though a standard t test on their and our data for means and standard deviations indicates a significant difference at the 0.0005 level between the two sets of results, the fact that their aortic and iliac axes were chosen subjectively can easily explain the variation. Raso et al.⁶ studied the aortograms of 28 women and 23 men without atherosclerotic involvement. They measured the angle between the aorta and the left common iliac artery and obtained a mean of 21° with a range of 7° to 35° for the men.

As shown in Table 1, we also found no significant difference in the branch angles between men and women. If bifurcation symmetry is assumed and the two groups of individuals are combined, the mean branch angle is 42° with a range of 15° to 90°, which is similar to the results from our study and that by Lee et al. Walburn et al.¹³ gave a mean of 51° with a standard deviation of 13° for 19 individuals (authors reported SEM) and a range of 37° to 80°, which is included in our range.

Sharp et al.⁴ reviewed 100 consecutive abdominal aortograms and found that patients with occlusive disease had an average bifurcation angle of 38°, and those with normal arteries or aneurysmal disease had an average angle of 52°. Branch angle was correlated with the position of the bifurcation in the abdomen. Narrower angles were associated with positions above the fourth lumbar vertebral body; broader angles were related to a position below this level. This correlation was also reported by Barker¹⁵ and by De Laurentis et al.¹⁶ If the claim by Sharp et al.⁴ is substantiated, namely, that there is a high incidence of occlusive disease in the group with more acute angles, this would be further evidence of the importance of arterial geometry in atherogenesis. However, it should be noted that the difference between the means of the two groups in the study by Sharp et al.⁴ is only 14°, which, when compared to the standard deviation of 12° reported by Lee et al.⁷ and confirmed here, indicates that the two distributions are probably broadly overlapping.

In view of the above results obtained by ourselves and others, the statement of Barker¹⁵ that the "normal bifurcation angle is 80° in humans" is probably incorrect. In our set of measurements, for example, none of the 70 bifurcations had an angle greater than 62°. There does not seem to exist a bias in our vessel selection process that would cause our branch angles to be different from "normal."

Although Caro et al.¹⁷ discuss dimensional asymmetry and its effect on shear rate at the aortic bifurcation, angular asymmetry and flow-divider offset have not generally been mentioned. Nevertheless, these geometric features have significant effects on the flow field near the bifurcation.¹ A flow divider that is offset from the aortic axis induces asymmetry in the flow field and strengthens secondary flows. In a branch with no concomitant angular asymmetry, the lateral aortic wall farther from the flow divider experiences an inordinately high shear stress, whereas the opposing lateral wall has a low shear stress. The flow in a branch with angular asymmetry alone is also strongly affected locally. However, as stated earlier, angular asymmetry is generally correlated with flow divider offset, so the shear distributions in real arteries will often reflect more than one of these geometric features.

There are other geometric features that influence the flow field but have not been dealt with there. As noted earlier, the gentle intrusion of an arterial wall into the flow as the divider is approached has important consequences. Also, the ratio of the sum of the cross-sectional areas of the iliac arteries to that of the aorta is significant and has received some attention.⁵,⁷,¹²,¹⁷ In the future, we hope to determine objectively the distributions of these and other geometric features among individuals.

### Table 1. Geometric Parameters of Human Aortic Bifurcations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Men</th>
<th>Women</th>
<th>Both</th>
<th>Male-female difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offset (mm)</td>
<td>-0.1 ± 2.6</td>
<td>+0.1 ± 1.4</td>
<td>0.02 ± 2.0</td>
<td>NS</td>
</tr>
<tr>
<td>Magnitude of offset (mm)</td>
<td>1.9 ± 1.9</td>
<td>1.0 ± 0.9</td>
<td>1.4 ± 1.4</td>
<td>p &lt; 0.01*</td>
</tr>
<tr>
<td>Branch angle (°)</td>
<td>34.2 ± 11.7</td>
<td>35.5 ± 10.6</td>
<td>35.0 ± 11.1</td>
<td>NS</td>
</tr>
<tr>
<td>Angular asymmetry (°)</td>
<td>5.5 ± 24.0</td>
<td>0.24 ± 16.1</td>
<td>2.2 ± 19.8</td>
<td>NS</td>
</tr>
<tr>
<td>Magnitude of angular asymmetry (°)</td>
<td>19.5 ± 14.9</td>
<td>12.4 ± 10.3</td>
<td>15.3 ± 12.9</td>
<td>p &lt; 0.025*</td>
</tr>
<tr>
<td>No. in group</td>
<td>28</td>
<td>42</td>
<td>70</td>
<td></td>
</tr>
</tbody>
</table>

Values are means ± sd.
* p is the probability that the difference in the male and female results occurred by chance.
GEOMETRIC PARAMETERS OF BIFURCATIONS

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Index Terms: arterial geometry • aortic bifurcation • branch angle • flow divider offset • angular asymmetry
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